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Research topics on Term Rewriting

Introduction to Term Rewriting: Techniques and Applications
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1 Modularity of confluence and termination

Toyama noticed that, in contrast to confluence [Toy87b], termination of term rewriting is *not* modular, even for TRSs that *do not share any function symbol* [Toy87a, TKB95]. Further developments in these topics by Toyama and others are summarized in [Ohl02, Chapter 8].

1. Main paper: [Toy87a].
2. Complementary paper: [Ohl02, Chapter 8].

2 Dependency pairs

State-of-the-art techniques and tools for proving termination of rewriting are based on exploiting the *dependency pairs approach* [AG00]. Further developments have lead to the so-called *dependency pairs framework* [GTSF06] which is the basis of modern tools for proving termination of term rewriting automatically.

1. Main paper: [AG00].
2. Complementary papers: [GTSF06] and [Ohl02, Section 5.4].

3 Synthesis of polynomial interpretations

Nowadays, polynomial interpretations are an essential ingredient in the development of tools for proving termination. Tools for automatically proving termination of rewriting *cannot* rely on user-defined polynomial interpretations for generating the polynomial orderings. How to *synthesize* appropriate polynomial orderings for a *given TRS*? A comprehensive account of this approach can be found here: [CMTU06].

1. Main paper: [CMTU06].
2. Complementary papers: [Luc07].

4 Conditional Term Rewriting Systems

Among the many possible extensions of the basic framework of term rewriting systems, conditional Term Rewriting Systems (CTRSs [DO90, Kap84]) play a fundamental role in algebraic specification of abstract data types and in rule-based programming languages such as Maude or Haskell.

1. Main paper: [DO90].
2. Complementary papers: [Ohl02, Chapter 7].

5 Maude

Maude [CDEL$^+$02, CDEL$^+$07] is a sophisticated specification and programming language whose operational principle is (essentially) term rewriting. Maude programs can be thought of as term rewriting systems enriched with a number of powerful features like *sorts*, *conditional equations and rules*, built-in datatypes (e.g., integers, real numbers), etc.

1. Main paper: [CDEL$^+$02].
2. Complementary papers: [CDEL$^+$07].

6 Matrix interpretations

Matrix interpretations [EWZ08] are a new kind of algebraic interpretations with interesting capabilities for proving termination of rewriting systems. Roughly speaking, a matrix interpretation for a k-ary symbol f is a linear expression $F_1x_1 + \cdots + F_kx_k + F_0$ where the $F_1, \ldots, F_k$ are matrices of $n \times n$ natural numbers and the variables $x_1, \ldots, x_k$ (and also F_0) represent n-tuples of natural numbers.

1. Main paper: [EWZ08].
2. Complementary papers: [HW06].

References

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